Q1: Let
$$\|x\|_1 = |x_1| + |x_2| + \cdots + |x_n|$$
for $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$.

Let A be any real $n \times n$ matrix. Prove that
$$\|Ax\|_1 \leq \|x\|_1 \cdot \max_{1 \leq l \leq n} \sum_{k=1}^{n} |A_{kl}|.$$

Hence prove that
$$\|A\|_1 = \max_{1 \leq l \leq n} \sum_{k=1}^{n} |A_{kl}|$$

Q2: Let $\|x\|_\infty = \max_{1 \leq k \leq n} |x_k|$

for $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$.

Prove that
$$\|Ax\|_\infty \leq \|x\|_\infty \max_{1 \leq k \leq n} \sum_{l=1}^{n} |A_{kl}|.$$

Hence prove that
$$\|A\|_\infty = \max_{1 \leq k \leq n} \sum_{l=1}^{n} |A_{kl}|.$$

Q3 : The FROBENIUS NORM of any real $n \times n$ matrix A is defined by

$$\|A\|_F = \sqrt{\sum_{k=1}^{n} \sum_{l=1}^{n} A_{kl}^2} \, .$$

Is this an operator norm?

EXTRA : Prove that

$$\|A\|_F^2 = \operatorname{trace} A^T A \, .$$