$$F^{(m)}_{jk} = \sum_{p=0}^{2^m-1} f\left(e^{2\pi i\left(\frac{p}{2^m}+\frac{k}{2^M}\right)}\right) e^{+2\pi i jp/2^m}$$

for $j = 0 : 2^m - 1$, $\quad k = 0 : 2^{M-m} - 1$, $\quad m = 0 : M$.

So $F^{(m)}$ is $2^m \times 2^{k-m}$

$F^{(0)} \quad 1 \times 2^M \qquad$ FUNCTION VALUES

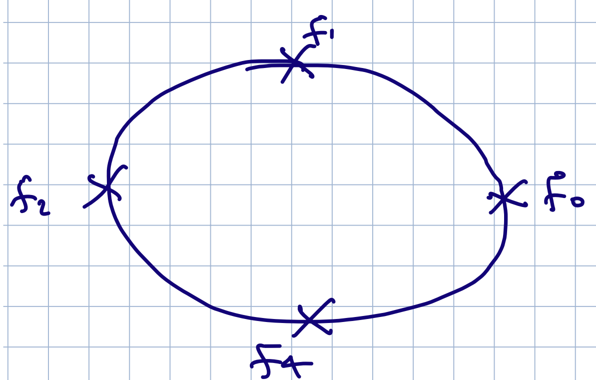
$F^{(1)} \quad 2 \times 2^{M-1}$

$F^{(2)} \quad 2^2 \times 2^{M-2}$

$\vdots$

$F^{(M)} \quad 2^M - 1 \qquad$ FFT

EXAMPLE $\quad M = 4$



$$F^{(0)} = (f_0 \quad f_1 \quad f_2 \quad f_3)$$

$$F^{(1)} = \begin{pmatrix} f_0+f_2 & f_1+f_3 \\ f_0-f_2 & f_1-f_3 \end{pmatrix}$$

KEY RESULT:

$$F^{(m)}_{jk} = F^{(m-1)}_{jk} + e^{+\pi i j/2^{m-1}} F^{(m-1)}_{j,\,k+2^{M-m}}.$$

$$F^{(1)} = \begin{pmatrix} f_0 + f_2 & f_1 + f_3 \\ f_0 - f_2 & f_1 - f_3 \end{pmatrix}$$

So

$m = 2 = M$

$$F^{(2)} = \begin{pmatrix} f_0 + f_2 + f_1 + f_3 \\ f_0 - f_2 + i(f_1 - f_3) \\ f_0 + f_2 - (f_1 + f_3) \\ f_0 - f_2 - i(f_1 - f_3) \end{pmatrix}$$

$$\begin{array}{ll} & j \\ e^{+\pi i j/2^{m-1}} = \begin{cases} 1 \\ i \\ -1 \\ -i \end{cases} & \begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array}$$

QUESTION 6:

$$\underset{1\times 8}{F^{(0)}} = (\overbrace{2 \quad 0 \quad 6 \quad -2 \quad 6}\quad 0 \quad 6 \quad 2)$$

$$\underset{2\times 4}{F^{(1)}} = \begin{pmatrix} \underset{2+6}{8} & 0 & \underset{6+6}{12} & 0 \\ \underset{2-6}{-4} & 0 & \underset{6-6}{0} & \underset{-2-2}{-4} \end{pmatrix}$$

$$\begin{array}{ll} & j \\ e^{\pi i j} = \begin{cases} 0 \\ 1 \end{cases} & \begin{array}{l} 0 \\ 1 \end{array} \end{array}$$

$$\underset{4\times 2}{F^{(2)}} = \begin{pmatrix} \underset{8+12}{20} & 0 \\ -4 & -4i \\ \underset{8-12}{-4} & 0 \\ -4 & 4i \end{pmatrix}$$

$$\begin{array}{ll} & j \\ e^{\pi i j/2} = \begin{cases} 1 \\ i \\ -1 \\ -i \end{cases} & \begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$\underset{8\times 1}{F^{(3)}} = \begin{pmatrix} 20 \\ -4-4i\alpha \\ -4 \\ -4-4\alpha \\ 20 \\ -4+4i\alpha \\ -4 \\ -4+4i\alpha^* \end{pmatrix}$$

ONLY EVENS

$$\alpha = e^{\pi i/4} = \begin{array}{cc} & j \\ \left\{\begin{array}{c} 1 \\ \alpha \\ i \\ i\alpha \\ -1 \\ -\alpha \\ -i \\ \alpha^* \end{array}\right. & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \end{array}$$

DIRECT CALCULATION: USE MATLAB!