14. $\mu = k^2/h^2$, $\quad u_{tt} = u_{xx} \Rightarrow -\tau^2 \hat{u} = -z^2 u$

$$\hat{R} = (e^{ik\tau} - 2 + e^{-ik\tau})$$
$$\qquad - \mu\left[a e^{ik\tau} + b + c e^{-ik\tau}\right](e^{ihz} - 2 + e^{-ihz})$$

$$= 2(\cos k\tau - 1) - \mu\left[a e^{ik\tau} + b + c e^{-ik\tau}\right] \cdot 2(\cos hz - 1).$$

Now $\quad \cos k\tau - 1 = -\frac{1}{2}k^2\tau^2 + \frac{1}{4!}k^4\tau^4 - \frac{1}{6!}k^6\tau^6 + \cdots$

$$= -\tfrac{1}{2}\mu h^2 z^2 + O(h^4).$$

Thus

$$\hat{R} = -\tfrac{1}{2}\mu h^2 z^2 + O(h^4)$$
$$\qquad - \mu\,\underbrace{\left[a e^{i\mu^{\frac{1}{2}}h\tau} + b + c e^{-i\mu^{\frac{1}{2}}h\tau}\right]}_{a+b+c \,+\, i\mu^{\frac{1}{2}}h\tau(a-c) \,+\, O(h^2)}\left(-\tfrac{1}{2}h^2z^2 + O(h^4)\right)$$

$$= -\tfrac{1}{2}\mu h^2 z^2 \left\{1 - (a+b+c)\right\}$$
$$\qquad + Ch^3(a-c) + O(h^4).$$

Hence $O(h^4)$ iff $\quad a+b+c = 1 \quad$ and $\quad a = c$,

i.e. $\quad b = 1 - 2a$

We have

$$\left[\begin{array}{l}\hat{g}_n[-2+4\mu s^2 b] \\ = -2\hat{g}_n(1-2\mu s^2 b)\end{array}\right]$$

$$\hat{g}_{n+1} - 2\hat{g}_n + \hat{g}_{n-1}$$

$$= \mu a \cdot 2(\cos\theta - 1)\hat{g}_{n+1}$$
$$+ \mu b \cdot 2(\cos\theta - 1)\hat{g}_n$$
$$+ \mu c \cdot 2(\cos\theta - 1)\hat{g}_{n-1} \qquad (c = a)$$

$$= \underbrace{2\mu(\cos\theta - 1)}_{-4\mu s^2}\left[a\hat{g}_{n+1} + \underset{1-2a}{\overset{b}{\shortparallel}}\hat{g}_n + a\hat{g}_{n-1}\right].$$

Then

$$\hat{g}_{n+1}\underbrace{(1+4\mu s^2 a)}_{\alpha} - 2\hat{g}_n\underbrace{(1-2\mu b s^2)}_{\beta} + \underbrace{(1+4\mu s^2 a)}_{\alpha}\hat{g}_{n-1} = 0$$

OR $r^2 - 2\nu r + 1 = 0$, $\nu = \beta/\alpha$.

Thus $r_1 r_2 = 1$. Further $(r-\nu)^2 + 1 - \nu^2 = 0$

or $r = \nu \pm \sqrt{(\nu^2 - 1)}$. Need complex conjugate pair for stability, i.e. $\nu^2 < 1$. [$\nu^2 = 1 \Rightarrow \nu = \pm 1$.]

Now $\nu^2 < 1$ iff $\beta^2 < \alpha^2$.

HINT: Now show that

$$\alpha^2 - \beta^2 \geq 0 \quad \text{iff} \quad 2a - b \geq 0.$$

Of course, $2a + b = 1$, so $2a + b \geq 0$ and thus

$2a \pm b \geq 0$, i.e. $2a \geq |b|$.