

26. I did this question using Matlab, which avoids much tedious calculation! Here are the details for $h = \frac{1}{6}$:

If

$$f(x,y) = 36(x^2 + y^2 - (x+y))$$

then

$$F(mh, nh) = m^2 + n^2 - (m+n) \quad (h = \frac{1}{6} \Rightarrow 36h^2 = 1)$$

and

$$\text{RESIDUAL} = \begin{pmatrix} -10 & -13 & -14 & -13 & -10 \\ -13 & -16 & -17 & -16 & -13 \\ -14 & -17 & -18 & -17 & -14 \\ -13 & -16 & -17 & -16 & -13 \\ -10 & -13 & -14 & -13 & -10 \end{pmatrix}.$$

$$\begin{aligned} \text{RESTRICTION} &= 4h^2 \left[-16 \cdot \frac{1}{4} + (-13 - 13 - 17 - 17) \cdot \frac{1}{8} \right. \\ &\quad \left. + (-10 - 14 - 14 - 18) \cdot \frac{1}{16} \right] \\ &= -60h^2 = -60/36 = -5/3. \end{aligned}$$

EXACT SOLUTION: $u_{20} + u_{24} + u_{02} + u_{42} - 4u_{22} = -5/3.$

TRICK: Symmetry $\Rightarrow$ solve $u + u - 4u = -5/3$
 $\Rightarrow u = 5/6.$

PROLONGATION:

$$\frac{1}{6} \begin{pmatrix} 5/4 & 5/2 & 5/2 & 5/2 & 5/4 \\ 5/2 & 5 & 5 & 5 & 5/2 \\ 5/2 & 5 & 5 & 5 & 5/2 \\ 5/2 & 5 & 5 & 5 & 5/2 \\ 5/4 & 5/2 & 5/2 & 5/2 & 5/4 \end{pmatrix}$$

$$\frac{5}{14} \quad \frac{5}{12} \quad \frac{5}{12} \quad \frac{5}{12} \quad \frac{5}{14} \quad / 1$$

JACOBI:

$$u_{33}^{(1)} = \frac{1}{4} \left[u_{32} + u_{34} + u_{23} + u_{43} - h^2 f_{33} \right]$$

$$= \frac{1}{4} \cdot 4 \cdot \frac{5}{6} + \frac{1}{36} \cdot 18 = \frac{5}{6} + \frac{1}{8} = \frac{23}{24}$$

RECOMMENDED: Use MATLAB first!